QUT Robots for the real world

Simultaneous Localization And Mapping - (1)

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Learning objectives

- SLAM using an extended Kalman filter.

Lecture 8 - 9 recap

Prediction step:

$$\bar{\mu}_t = f(\mu_{t-1}, \mathbf{u}_t)$$

$$\bar{\Sigma}_t = \mathbf{J}_{x_t} \mathbf{\Sigma}_{t-1} \mathbf{J}_{x_t}^T + \mathbf{J}_{u_t} \mathbf{R} \mathbf{J}_{u_t}^T$$

Update step:

For each observed landmark do:

$$\bar{\mu}_t = \bar{\mu}_t + \mathbf{K}_t^i (\mathbf{z}_t^i - h(\bar{\mu}_t, i))$$

$$\bar{\mathbf{\Sigma}}_t = (\mathbf{I} - \mathbf{K}_t^i \mathbf{G}_t^i) \bar{\mathbf{\Sigma}}_t$$

Prediction step:

$$\bar{\mu}_t = \mu_{t-1}$$

$$\bar{\mathbf{\Sigma}}_t = \mathbf{\Sigma}_{t-1}$$

Update step:

For each observed landmark do:

$$\bar{\mu}_t = \bar{\mu}_t + \mathbf{K}_t^i (\mathbf{z}_t^i - h(\bar{\mu}_t, i, \mathbf{x}_t^r))$$

$$\bar{\mathbf{\Sigma}}_t = (\mathbf{I} - \mathbf{K}_t^i \mathbf{G}_t^i) \bar{\mathbf{\Sigma}}_t$$

Assumptions

- The robot does not know its pose in the map.
- The wheel encoders are noisy.
- The robot does not know the position of the landmarks in the map.
- The sensor onboard the robot is noisy.
- The robot can associate the measurements with the landmarks.

The task

- The robot should **localize itself** inside a map using a set of landmarks and at the same time use its pose and sensor **to map** the positions of the landmarks.

SLAM: the chicken or egg problem

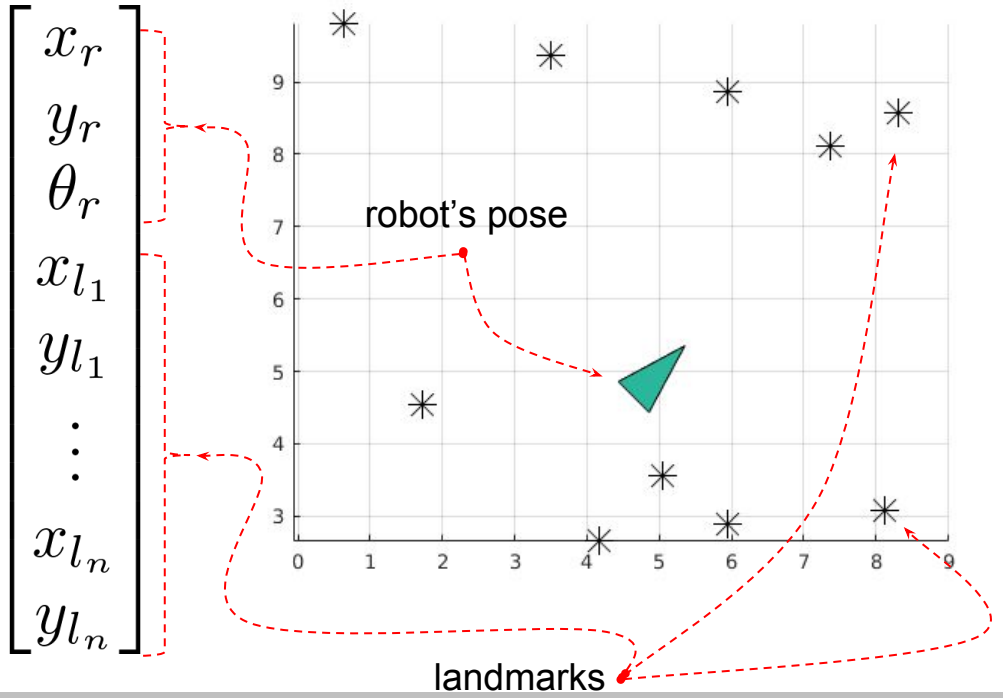
- As we saw in lecture 8, we need the position of the landmarks (i.e the map) to estimate the pose of the robot.
- And we saw in lecture 9 that in order to estimate the position of the landmarks in the map we need the true pose of the robot.
- In this lecture we are going to do the two above processes at the same time. This is called simultaneous localisation and mapping (SLAM).

Localize yourself in a map that you are building using the estimation of your pose in it!

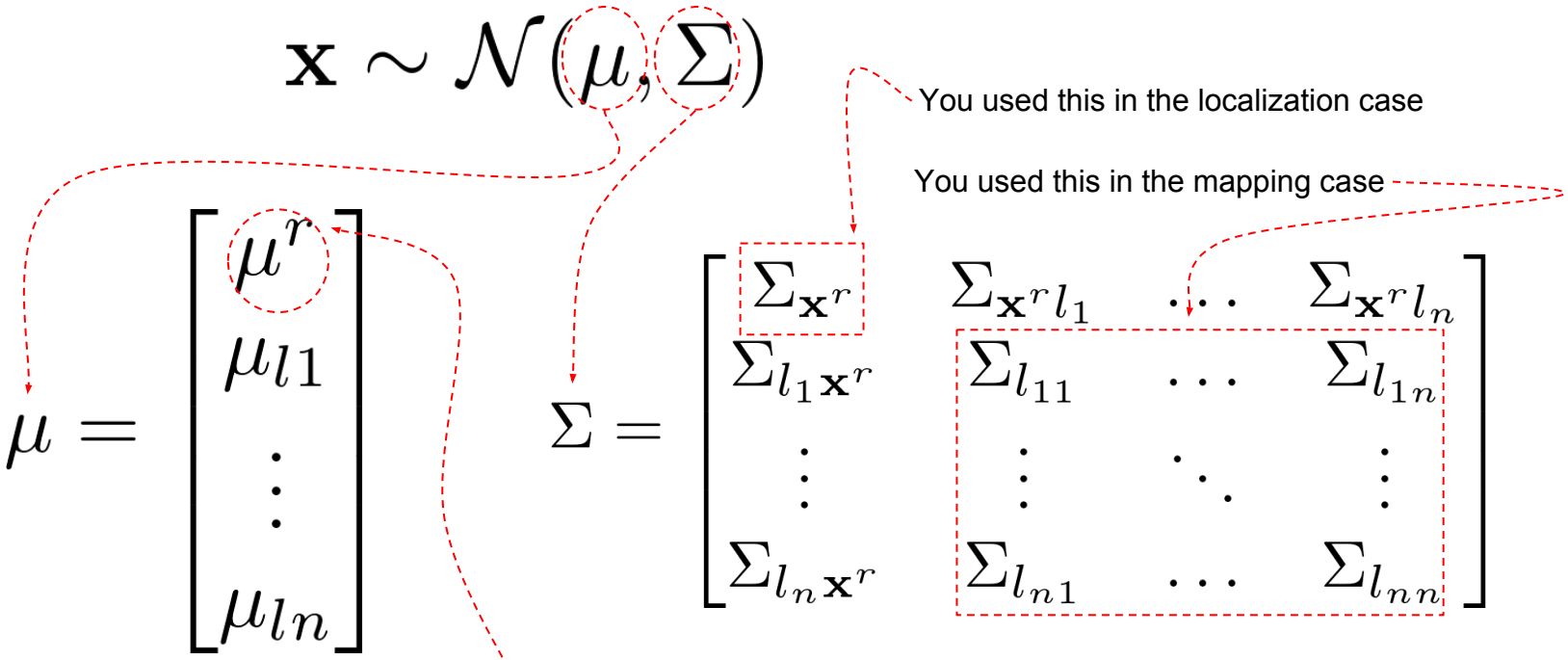
The state vector

- The state vector contains both the pose of the robot and the positions of the landmarks in the map.

$$\mathbf{x}_t = \begin{bmatrix} \mathbf{x}^r \\ M \end{bmatrix} = \begin{bmatrix} x_r \\ y_r \\ \theta_r \\ x_{l_1} \\ y_{l_1} \\ \vdots \\ x_{l_n} \\ y_{l_n} \end{bmatrix}$$



We still live in a Gaussian world!

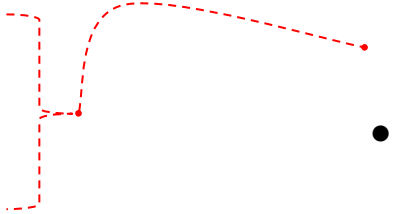


The same set of equations

Prediction step:

$$\bar{\mu}_t = f(\mu_{t-1}, \mathbf{u}_t)$$

$$\bar{\Sigma}_t = \mathbf{J}_{x_t} \mathbf{\Sigma}_{t-1} \mathbf{J}_{x_t}^T + \mathbf{J}_{u_t} \mathbf{R} \mathbf{J}_{u_t}^T$$



- We predict both the pose of the robot and the positions of the landmarks.
- In the prediction step the robot moves and the landmarks stay static.

Update step:

For each landmark $\mathbf{z}_t^i$ do:

$$\mu_t = \bar{\mu}_t + \mathbf{K}_t^i (\mathbf{z}_t^i - h(\bar{\mu}_t, i))$$

$$\mathbf{\Sigma}_t = (\mathbf{I} - \mathbf{K}_t^i \mathbf{G}_t^i) \bar{\mathbf{\Sigma}}_t$$

end

$$\mu_t = \bar{\mu}_t$$

$$\mathbf{\Sigma}_t = \bar{\mathbf{\Sigma}}_t$$

$$\mathbf{K}_t = \bar{\mathbf{\Sigma}}_t \mathbf{G}_t^T (\mathbf{G}_t \bar{\mathbf{\Sigma}}_t \mathbf{G}_t^T + \mathbf{Q})^{-1}$$

Prediction step:

$$\bar{\mu}_t = \begin{bmatrix} f_r(\mu^r_{t-1}, \mathbf{u}_t) \\ \mu_{l1_{t-1}} \\ \vdots \\ \mu_{ln_{t-1}} \end{bmatrix}$$

$$\bar{\Sigma}_t = \mathbf{J}_{x_t} \mathbf{\Sigma}_{t-1} \mathbf{J}^T_{x_t} + \mathbf{J}_{u_t} \mathbf{R} \mathbf{J}^T_{u_t}$$

$\mathbf{J}_{x_t}$ The Jacobian matrix of **f** w.r.t the state vector.

$\mathbf{J}_{u_t}$ The Jacobian matrix of **f** w.r.t the odometry.

If at time step **t** we have mapped **n** landmarks, what is the dimension of these matrices?

The prediction step

The same matrices from the localization case

$$\mathbf{J}_{x_t} = \begin{bmatrix} \mathbf{J}_{x_t^r} & 0_{3\times 2n} \\ 0_{2n\times 3} & I_{2n\times 2n} \end{bmatrix} \qquad \mathbf{J}_{u_t} = \begin{bmatrix} \mathbf{J}_{u_t^r} \\ 0_{2n\times 2} \end{bmatrix}$$

$$\bar{\Sigma}_t = \mathbf{J}_{x_t}\boldsymbol{\Sigma}_{t-1}\mathbf{J}_{x_t}^T + \mathbf{J}_{u_t}\mathbf{R}\mathbf{J}_{u_t}^T$$

$$\bar{\Sigma}_t = \mathbf{J}_{x_t} \boldsymbol{\Sigma}_{t-1} \mathbf{J}_{x_t}^T + \mathbf{J}_{u_t} \mathbf{R} \mathbf{J}_{u_t}^T$$

$$\begin{bmatrix} \mathbf{J}_{x_t^r} & 0_{3\times 2n} \\ 0_{2n\times 3} & I_{2n\times 2n} \end{bmatrix} \begin{bmatrix} \Sigma_{\mathbf{x}^r} & \Sigma_{\mathbf{x}^r l_1} & \cdots & \Sigma_{\mathbf{x}^r l_n} \\ \Sigma_{l_1 \mathbf{x}^r} & \Sigma_{l_{11}} & \cdots & \Sigma_{l_{1n}} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{l_n \mathbf{x}^r} & \Sigma_{l_{n1}} & \cdots & \Sigma_{l_{nn}} \end{bmatrix} \begin{bmatrix} \mathbf{J}_{x_t^r} & 0_{3\times 2n} \\ 0_{2n\times 3} & I_{2n\times 2n} \end{bmatrix}^T$$

We use the same treatment with this term as well.

The same set of equations

Prediction step:

$$\bar{\mu}_t = f(\mu_{t-1}, \mathbf{u}_t)$$

$$\bar{\Sigma}_t = \mathbf{J}_{x_t} \mathbf{\Sigma}_{t-1} \mathbf{J}_{x_t}^T + \mathbf{J}_{u_t} \mathbf{R} \mathbf{J}_{u_t}^T$$

Update step:

For each landmark $\mathbf{z}_t^i$ do:

$$\mu_t = \bar{\mu}_t + \mathbf{K}_t^i(\mathbf{z}_t^i - h(\bar{\mu}_t, i))$$

$$\mathbf{\Sigma}_t = (\mathbf{I} - \mathbf{K}_t^i \mathbf{G}_t^i)\bar{\mathbf{\Sigma}}_t$$

end

$$\mu_t = \bar{\mu}_t$$

$$\mathbf{\Sigma}_t = \bar{\mathbf{\Sigma}}_t$$

Similar to the mapping case but with the fact that the pose of the robot is now part of the state vector

$$\mathbf{K}_t = \bar{\mathbf{\Sigma}}_t \mathbf{G}_t^T (\mathbf{G}_t \bar{\mathbf{\Sigma}}_t \mathbf{G}_t^T + \mathbf{Q})^{-1}$$

The same measurement function we used for localization and for mapping.

$$h(\bar{\mu}_t, i) = \begin{bmatrix} r^i \\ \beta^i \end{bmatrix} = \begin{bmatrix} \sqrt{(x_r - x_{l_i})^2 + (y_r - y_{l_i})^2} \\ atan2(y_{l_i} - y_r, x_{l_i} - x_r) - \theta_r \end{bmatrix}$$

Range and bearing with respect to robot's own frame of reference at time step t.

Coordinates of landmark i from the vector $\bar{\mu}_t$

The predicted pose of the robot from the vector $\bar{\mu}_t$

The Jacobian matrix of the measurement function

$$\mathbf{G}_t^i = \frac{\partial h(\bar{\mu}_t, i)}{\partial \bar{\mu}_t}$$

$$= \begin{bmatrix} -\frac{x_{l_i} - x_r}{r} & -\frac{y_{l_i} - y_r}{r} & 0 & \cdots & \frac{x_{l_i} - x_r}{r} & \frac{y_{l_i} - y_r}{r} & \cdots \\ \frac{y_{l_i} - y_r}{r^2} & -\frac{x_{l_i} - x_r}{r^2} & -1 & \cdots & -\frac{y_{l_i} - y_r}{r^2} & \frac{x_{l_i} - x_r}{r^2} & \cdots \end{bmatrix}$$

zeros

The same set of equations

Prediction step:

$$\bar{\mu}_t = f(\mu_{t-1}, \mathbf{u}_t)$$

$$\bar{\Sigma}_t = \mathbf{J}_{x_t} \mathbf{\Sigma}_{t-1} \mathbf{J}_{x_t}^T + \mathbf{J}_{u_t} \mathbf{R} \mathbf{J}_{u_t}^T$$

Update step:

For each landmark $\mathbf{z}_t^i$ do:

$$\bar{\mu}_t = \bar{\mu}_t + \mathbf{K}_t(\mathbf{z}_t^i - h(\bar{\mu}_t, i))$$

$$\bar{\mathbf{\Sigma}}_t = (\mathbf{I} - \mathbf{K}_t^i \mathbf{G}_t^i)\bar{\mathbf{\Sigma}}_t$$

end

$$\mu_t = \bar{\mu}_t$$

$$\mathbf{\Sigma}_t = \bar{\mathbf{\Sigma}}_t$$

But what if we see a landmark for the first time?

$$\mathbf{K}_t = \bar{\mathbf{\Sigma}}_t \mathbf{G}_t^T (\mathbf{G}_t \bar{\mathbf{\Sigma}}_t \mathbf{G}_t^T + \mathbf{Q})^{-1}$$

Landmark initialization

$$\bar{\mu}_t{}^* = \begin{bmatrix} \bar{\mu}_t \\ l_{new} \end{bmatrix} = \begin{bmatrix} \bar{\mu}_t \\ x_{l_{new}} \\ y_{l_{new}} \end{bmatrix}$$

You already know how to find these as we already encountered them in the mapping case during last lecture.

Simply expand the state vector with the coordinates of the new landmark in the map!

The landmark initialisation function

$$\mathbf{z} = \begin{bmatrix} r \\ \beta \end{bmatrix}$$

Sensor measurement to a never seen before landmark.

$$l_{new} = q(\mathbf{z}_t^{new}, \bar{\mu}_t)$$

$$l_{new} = \begin{bmatrix} x_r + r \times cos(\theta_r + \beta) \\ y_r + r \times sin(\theta_r + \beta) \end{bmatrix}$$

The first three components of $\bar{\mu}_t$

The Jacobian of the landmark initialisation function w.r.t the **z**

$$\mathbf{L}_z = \frac{\partial q(\mathbf{z}, \bar{\mu}_t)}{\partial \mathbf{z}}$$

$$= \begin{bmatrix} cos(\theta_r + \beta) & -r \times sin(\theta_r + \beta) \\ sin(\theta_r + \beta) & r \times cos(\theta_r + \beta) \end{bmatrix}$$

What about the covariance matrix?

$$\bar{\Sigma}_t^* = \begin{bmatrix} \bar{\Sigma}_t & 0 \\ 0 & \mathbf{L_z Q L_z^T} \end{bmatrix}$$

The covariance matrix expands as well!

Zero matrices!

The covariance of the sensor noise.

Putting it all together

1. Move.
2. Perform the **prediction step** which updates the mean and covariance.
3. Make a new Measurement (range and bearing to a landmark).
4. if we have not seen the landmark before:
    - Do landmark initialization based on the robot estimated pose.

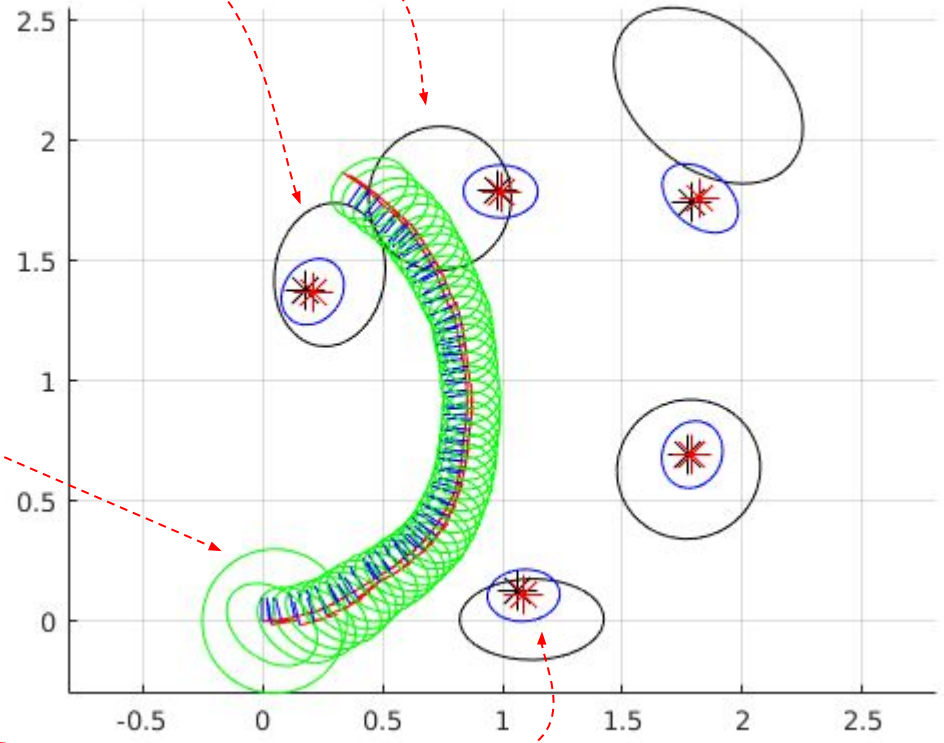
   else

    - Predict the landmark position based on the robot estimated pose.
5. Perform the **update step** and update the mean and the covariance.
6. Go to 1.

The uncertainty on the positions of the landmarks on initialization.

The uncertainty on the pose of the robot (green ellipses)

The uncertainty on the position of the landmarks after 50 steps (**blue ellipses**)



Next Lecture

What are the strengths and weaknesses of EKF-SLAM and what are the other flavours of SLAM algorithms?